

## International Research Network Project "SYMMETRY, TOPOLOGY and MODULI"

# The 11th OCAMI-RIRCM Joint Differential Geometry Workshop on Submanifolds and Lie Theory

**Date:** March 20 (Sun)- 23 (Wed), 2016

**Place:** Lecture Room E408, Department of Mathematics, [Graduate School of Science, Osaka City University](#)

**Organizers:** Yoshihiro Ohnita (OCU, Director of OCAMI, Japan), Young Jin Suh (KNU, Director of RIRCM, Korea), Juergen Berndt (Kings College London, UK), Hyunjin Lee (KNU & RIRCM, Korea)

**Sponsors :** Osaka City University Advanced Mathematical Institute (OCAMI),  
Research Institute of Real and Complex Manifolds (RIRCM) & Department of Mathematics,  
Kyungpook National University (KNU),

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**Invited speakers :** Professor Jost-Hinrich Eschenburg (University of Augsburg, Germany)  
Professor Marcos Alexandrino (Institute of Mathematics and Statistics, University of Sao Paulo, Brasil)  
Professor Leonardo Biliotti (University of Parma, Italy)  
Professor Juergen Berndt (King's College London, UK)  
Professor Young Jin Suh (Kyungpook National University, Korea)  
Professor Hiroyuki Tasaki (University of Tsukuba, Japan)  
Professor Makiko Sumi Tanaka (Tokyo University of Science, Japan)  
Professor Takashi Sakai (Tokyo Metropolitan University, Japan)  
Professor Naoyuki Koike (Tokyo University of Science, Japan)  
Professor Hiroshi Tamaru (Hiroshima University, Japan)  
Professor Takayuki Okuda (Hiroshima University, Japan)  
Doctor Hyunjin Lee (Kyungpook National University, Korea)  
Doctor Eunmi Pak (Kyungpook National University, Korea)  
Mr. Changhwa Woo (Kyungpook National University, Korea)  
Mr. Doo Hyun Hwang (Kyungpook National University, Korea)  
Mr. Kyujong Kim (Kyungpook National University, Korea)  
Mr. Shinji Ohno (Tokyo Metropolitan University, Japan)

etc.

## Title and Abstract of Talks : [PDF](#)

Jost-Hinrich Eschenburg

Title: Submanifolds and Bott Periodicity (Joint work with Bernhard Hanke)

Abstract: Some compact symmetric spaces have poles and equators like spheres. Poles are points with the same geodesic symmetry, and equators contain the midpoints of shortest geodesics between poles. These are reflective submanifolds called centrioles. Some centrioles have topological meaning as homotopy approximations of the full path space, thus iterated centrioles serve to compute higher homotopy groups. This is the basic idea of Milnor's version of Bott's periodicity theorem. We want to show that this idea can be applied also to the vector bundle versions of the periodicity theorem. As an example, we will talk about a new proof for the theorem of Atiyah, Bott, and Shapiro (1964) which relates vector bundles over spheres to Clifford modules; these in turn are related to iterated centrioles of the classical groups. (Common work with Bernhard Hanke, Augsburg)

Title: What is a Penrose tiling? (Special Talk)

Abstract: As finite beings with only a finite set of ideas in mind, how can we invent tilings covering the whole infinite plane (complete tilings)? In art, two answers have been developed: Periodicity (a finite domain repeats itself in two directions) and Self Similarity (a small area determines the pattern for a larger area). In 1974, Roger Penrose introduced a class of tilings which are not periodic but have a certain self similarity property. They are constructed from two types of tiles, based on the geometry of the regular pentagon. So far there is no valid constructive definition for Penrose tilings which are extendible to complete ones. Matching rules (see Wikipedia article on Penrose tilings) are not sufficient as we will see. One way to construct complete tilings is by projecting a strip of a higher dimensional regular grid. However, we will give an example of a Penrose tiling which does not arise in this way. We use an extremely narrow definition of Penrose tilings which seems to completely determine those patterns, but still it allows uncountably many different patterns, including all projection tilings. This finding was heavily influenced by some 400 years old Islamic patterns at Isfahan, Iran. (Common work with H.J. Rivertz, Trondheim, Norway)

Marcos Alexandrino (1)

Title: Mean curvature flow of orbits of isometric actions

Abstract: Given a proper isometric action on a compact manifold, we study the mean curvature flow equation with a principal orbit of this action as initial datum. We prove that any finite time singularity is a singular orbit, and the singularity is of type I. These results are proved in the more general context of Singular Riemannian foliations and generalize previous results of Liu and Terng, Pacini and Koike. This talk is based on a joint work with Dr. Marco Radeschi (wwu- Munster) and is aimed at a broad audience of students, faculties and researchers. M.M. Alexandrino and M. Radeschi, Mean curvature flow of singular Riemannian foliations, to appear in The Journal of Geometric Analysis (2015) see also arXiv:1408.543

Marcos Alexandrino (2)

Title: Isometry flows on orbit spaces

Abstract: In this talk, we discuss the following result: Given a proper isometric action  $S^1 \times M \rightarrow M$  on a complete Riemannian manifold  $M$  then each continuous isometric flow on the orbit space  $M/S^1$  is smooth, i.e., it is the projection of an  $S^1$ -equivariant smooth flow on the manifold  $M$ . The first application of our result concerns Molino's conjecture, which states that the partition of a Riemannian manifold into the closures of the leaves of a singular Riemannian foliation is still a singular Riemannian foliation. We prove Molino's conjecture for the main class of foliations considered in his book, namely orbit-like foliations. We also discuss smoothness of isometric actions on orbit spaces. This talk is based on a joint work with Dr. Marco Radeschi (wwu- Munster) and is aimed at a broad audience of students, faculties and researchers in Geometry. ps: I will also briefly explain current works and how this talk is related to holonomy foliation that is associated to every connections on fiber bundles.

Leonardo Biliotti (University of Parma, Italy)

Title: Stability of measures on Kähler manifolds

Abstract: Let  $M$  be a Kähler manifold and let  $K$  be a compact group that acts on  $M$  in a Hamiltonian fashion. We will study the action of  $K$  and its complexification  $G$  on the space of probability measures on  $M$ . First of all we identify an abstract setting for the momentum mapping and give numerical criteria for stability, semi-stability and polystability. Next we apply this setting to the action of  $G$  on measures. We get various stability criteria for measures on Kähler manifolds. The same circle of ideas gives a very general surjectivity result for a map originally studied by Hersch and Bourguignon-Li-Yau. This is a joint work with Dott. Alessandro Ghigi.

Juergen Berndt (King's College London, UK)

Title: Compact homogeneous Riemannian manifolds with low co-index of symmetry

Abstract: The co-index of symmetry measures in how far a homogeneous Riemannian manifold deviates from being a Riemannian symmetric space. In the talk we will discuss some general structure theory for compact homogeneous Riemannian manifolds in relation to the co-index of symmetry. We also present a classification of compact homogeneous Riemannian manifolds with co-index of symmetry less or equal than three. This is joint work with Carlos Olmos (Cordoba) and Silvio Reggiani (Rosario), to appear in Journal of the European Mathematical Society (see also arXiv:1312.6097).

Young Jin Suh (Kyungpook National University, Korea)

Title: Recent progress on complex quadrics in Hermitian Symmetric Spaces

Abstract: In this talk we will give some background on the geometry of complex quadric  $Q^m = SO(m+2)/SO(2) \times SO(m)$  which can be regarded as Hermitian Symmetric Space with rank 2 of compact type. We will give some detailed explanations about the progress on real hypersurfaces in the complex quadric  $Q^m$  for Ricci parallel, harmonic curvature, normal Jacobi operator, pseudo-Einstein real hypersurfaces, and pseudo-anti commuting and Ricci soliton problem etc.

Jong Ryul Kim (Kunsan National University)

Title: A gradient Ricci soliton with a regular hypersurface of a potential function  $f$  and  $\text{Hess}f(\nabla f, \nabla f) = 0$

Abstract: For a complete oriented Riemannian manifold  $M$  of a gradient Ricci soliton with a regular level hypersurface  $H_s = \{x \in M \mid f(x) = s\}$  for a potential function  $f$  with a unit normal vector field  $N = \frac{\nabla f}{|\nabla f|}$  on  $H_s$ , we show that if  $\text{Ric}(N, N) = \lambda$  and  $D_{E_i} R = 2 \text{Ric}(\nabla f, E_i)$  is constant for a local geodesic frame  $\{E_i\}_{i=1}^{n-1}$  of  $(\nabla f)^\perp$ , then  $M$  is Ricci flat and each level hypersurface is totally geodesic. Moreover  $M$  is a product manifold  $H_s \times \mathbb{R}$ .

Hiroyuki Tasaki (University of Tsukuba)

Title: Maximal antipodal sets of oriented real Grassmann manifolds

Abstract: An antipodal set in a compact Riemannian symmetric space is a subset where the restriction of the symmetry at each point is the identity, which was introduced by Chen and Nagano. A maximal antipodal set is a kind of frame of a compact Riemannian symmetric space. In this talk I mainly treat maximal antipodal sets of oriented real Grassmann manifolds. We can reduce the classification of these to a combinatorial problem and classify these in the case where the rank is less than five. Moreover we can estimate the cardinalities of maximal antipodal sets in cases of higher rank.

Makiko Sumi Tanaka (Tokyo University of Science)

Title: Antipodal sets of compact Riemannian symmetric spaces

Abstract: A subset of a compact Riemannian symmetric space is called an antipodal set if the geodesic symmetry at each point is the identity on the set. The cardinalities of antipodal sets are finite. An antipodal set whose cardinality attains the maximum of the cardinalities of antipodal sets is called a great antipodal set. In a symmetric  $R$ -space, a maximal antipodal set is a great antipodal set and a great antipodal set is unique up to the action of the identity component of the isometry group, moreover, a great antipodal set is an orbit of the Weyl group. On the other hand, we have not known much about antipodal sets of a compact Riemannian symmetric space which is not a symmetric  $R$ -space. In this talk we will present a classification of maximal antipodal subgroups in a quotient group of a compact Lie group of classical type and that in  $G_2$ . In many of these cases there exists a maximal antipodal subgroup but not a great antipodal subgroup. This talk is based on

joint work with Hiroyuki Tasaki.

Takashi Sakai (Tokyo Metropolitan University)

Title: Biharmonic homogeneous submanifolds in compact symmetric spaces

Abstract: In 1983, J. Eells and L. Lemaire extended the notion of harmonic map between Riemannian manifolds to that of biharmonic map, which is defined as a critical point of the bienergy functional. G.Y. Jiang studied the first and second variation formulas of the bienergy functional and obtained the Euler-Lagrange equation, which is a fourth order PDE. In this talk, first we study biharmonic submanifolds in Einstein manifolds, and give a necessary and sufficient condition for a submanifold whose tension field is parallel to be biharmonic. For orbits of commutative Hermann actions in compact symmetric spaces, this condition can be described in terms of symmetric triads. By using this criterion, we construct some proper biharmonic submanifolds in compact symmetric spaces, which are orbits of commutative Hermann actions. Here, proper biharmonic means that biharmonic, but not harmonic. This talk is based on a joint work with Shinji Ohno and Hajime Urakawa.

Naoyuki Koike (Tokyo University of Science)

Title: Volume-preserving mean curvature flow for tubes in symmetric spaces

Abstract: In this talk, we first state the evolutions of the radius function and its gradient along the volume-preserving mean curvature flow starting from a tube (of nonconstant radius) over a compact closed domain of a reflective submanifold in a symmetric space under certain condition for the radius function. Next, we prove that the tubeness is preserved along the flow in the case where the ambient space is a rank one symmetric space of compact type (other than a sphere), the reflective submanifold is an invariant submanifold and the initial radius function is radial. Furthermore, in this case, we prove that the flow exists in infinite time or reaches to one of the invariant submanifold, a focal submanifold of the invariant submanifold and the cut locus of the intersection point in a normal umbrella of the invariant submanifold.

Hiroshi Tamaru (Hiroshima University)

Title: Realizations of some contact metric manifolds as Ricci soliton real hypersurfaces

Abstract: Ricci soliton contact metric manifolds with some nullity conditions have recently been studied by Ghosh and Sharma. Whereas the gradient case is well-understood, they provided a list of candidates for the non-gradient case. These candidates can be realized as Lie groups, but one only knows the bracket relations of the Lie algebras, which are hard to be understood apart from the three-dimensional case. In this talk, we will study these spaces with higher-dimensions, and prove that the simply-connected ones can be realized as homogeneous real hypersurfaces in noncompact two-plane Grassmannians. These realizations enable us to prove, in a Lie-theoretic way, that all of them are actually Ricci solitons. This talk is based on a joint work with Jong Taek Cho, Takahiro Hashinaga, Akira Kubo, and Yuichiro Taketomi.

Takayuki Okuda (Hiroshima University)

Title: Pairs of conjugacy classes of reflective submanifolds of Riemannian symmetric spaces with discrete intersections

Abstract: Let  $M = G/K$  be a Riemannian symmetric space of compact or non-compact type. In this talk, we classify pairs of reflective submanifolds  $L_1$  and  $L_2$  in  $M$  such that the intersection of  $L_1$  and  $gL_2$  is discrete in  $M$  for any  $g$  in  $G$ . We will also talk about relations between such pairs  $(L_1, L_2)$  and discontinuous groups for certain pseudo-Riemannian symmetric spaces.

Hyunjin Lee and Young Jin Suh (Kyungpook National University, Korea)

Title: Some parallel Hopf hypersurfaces in complex hyperbolic two-plane Grassmannians

Abstract: In this talk, we first consider a concept named the cyclic parallelism for the shape operator of a real hypersurface  $M$  in complex hyperbolic two-plane Grassmannian. Moreover, we prove that the cyclic parallelism is to be a Reeb parallelism on  $M$ . Hence, we study the Reeb parallelism by the Levi-Civita and generalized Tanaka-Webster connections on  $M$ . Related to these notions, we give some characterizations of Hopf hypersurfaces in complex hyperbolic two-plane Grassmannians.

Eunmi Pak and Young Jin Suh (Kyungpook National University, Korea)

Title: Results on Jacobi operators of real hypersurfaces in complex two-plane Grassmannians

**Abstract:** In relation to the generalized Tanaka-Webster connection, we consider a new notion of parallel Jacobi operator for real hypersurfaces in complex two-plane Grassmannians and show results about real hypersurfaces in complex two-plane Grassmannians with generalized Tanaka-Webster parallel structure Jacobi operator and normal Jacobi operator.

Changhwa Woo, Hyunjin Lee and Young Jin Suh (Kyungpook National University, Korea)

**Title:** Real hypersurfaces in complex hyperbolic two-plane Grassmannians with commuting restricted structure Jacobi operators

**Abstract:** In this paper, we introduce a new commuting condition between the structure Jacobi operator and symmetric  $(1,1)$ -type tensor field  $T$ , that is,  $R_{\xi}\phi T = TR_{\xi}\phi$ , where  $T=A$  or  $T=S$  for Hopf hypersurfaces in complex hyperbolic two-plane Grassmannians. By using simultaneous diagonalization for commuting symmetric operators, we give a complete classification of real hypersurfaces in complex hyperbolic two-plane Grassmannians with commuting condition respectively.

Doo Hyun Hwang and Young Jin Suh (Kyungpook National University, Korea)

**Title:** Hopf hypersurfaces in complex two plane Grassmannians satisfying recurrent Ricci tensors

**Abstract:** In this paper, we have introduced a new notion of generalized Tanaka-Webster Reeb recurrent Ricci tensor of real hypersurfaces in complex two-plane Grassmannians. Next, we show a non-existence property for real hypersurfaces  $M$  in complex two-plane Grassmannians with such a curvature condition.

Gyu Jong Kim and Young Jin Suh (Kyungpook National University, Korea)

**Title:** Real hypersurfaces in complex two-plane Grassmannians with Killing structure Jacobi operator

**Abstract:** In this talk, we introduce a notion of the complex two-plane Grassmannians and a geometric meaning of Killing structure Jacobi operator. Then we give a non-existence theorem for a real hypersurface in complex two-plane Grassmannians satisfies the condition of Killing structure Jacobi operator.

Shinji Ohno (Tokyo Metropolitan University)

**Title:** A construction of weakly reflective submanifolds in compact symmetric spaces

**Abstract:** A submanifold of a Riemannian manifold is called a weakly reflective submanifold if for any point in the submanifold and each normal vector at the point, there exists a weakly reflection which is an isometry on the ambient Riemannian manifold. The notion of weakly reflective submanifold is a generalization of the notion of reflective submanifold. Reflective submanifolds in irreducible compact symmetric spaces are classified. In contrast, examples of weakly reflective submanifolds in compact symmetric spaces are not well known except for examples in hyperspheres. In this talk we will give examples of weakly reflective submanifolds in compact symmetric spaces as orbits of Hermann actions which are generalizations of isotropy actions of compact symmetric spaces. To construct weakly reflective submanifolds in compact symmetric spaces, we use symmetric triads which are generalizations of restricted root systems.

Yoshihiro Ohnita (OCAMI, Japan)

**Title:** Hamiltonian non-displaceability of Gauss images of isoparametric hypersurfaces (Joint work with Hiroshi Iriyeh, Hui Ma and Reiko Miyaoka,)

**Abstract:** In this talk we discuss the Hamiltonian non-displaceability of Gauss images of isoparametric hypersurfaces in the spheres as Lagrangian submanifolds embedded in complex hyperquadrics.

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**Program (provisional) :** [PDF](#)

3/20 (Sun) AM 10:00-11:00 Lecture Room E408 Young Jin Suh  
AM 11:10-12:10 Lecture Room E408 Jost-Hinrich Eschenburg  
PM 13:30-14:30 Lecture Room E408 Naoyuki Koike  
PM 14:40-15:40 Lecture Room E408 Marcos Alexandrino (1)

PM 15:50-16:50 Lecture Room E408 Takayuki Okuda  
PM 17:00-18:00 Lecture Room E408 Juergen Berndt  
Party

- 3/21 (Mon) AM 9:30-10:30 Lecture Room E408 Jong Ryul Kim  
AM 10:40-11:40 Lecture Room E408 Hiroshi Tamaru  
PM 13:00-13:30 Lecture Room F405 Shinji Ohno  
PM 13:35-14:05 Lecture Room F405 Hyunjin Lee  
PM 14:10-14:40 Lecture Room F405 Eunmi Pak  
PM 14:45-15:15 Lecture Room F405 Changhwa Woo  
PM 15:20-15:50 Lecture Room F405 Doo Hyun Hwang  
PM 15:55-16:25 Lecture Room F405 Kyujong Kim  
PM 17:00-17:50 Lecture Room F405 Jost-Hinrich Eschenburg (Special Talk)
- 3/22 (Tue) AM 10:00-11:00 Lecture Room E408 Hiroyuki Tasaki  
AM 11:10-12:10 Lecture Room E408 Makiko Sumi Tanaka  
PM 13:30-14:30 Lecture Room E408 Leonardo Billotti  
PM 14:40-15:40 Lecture Room E408 Marcos Alexandrino (2)  
PM 15:50-16:50 Lecture Room E408
- 3/23 (Wed) AM 10:00-11:00 Lecture Room E408 Takashi Sakai  
AM 11:10-12:10 Lecture Room E408 Yoshihiro Ohnita

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**Suggestion to Speakers** : At the lecture room there are enough blackboards, the computer projector and the visualizer. Please prepare your talk using them.

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**Link** : [ICM 2014 Satellite Conference on Real and Complex Submanifolds](#)  
[Osaka City University Advanced Mathematical Institute \(OCAMI\)](#)  
[Department of Mathematics, Osaka City University](#)  
[The 1st OCAMI-KNUGRG Joint International Workshop on Differential Geometry and Related Fields \(Oct. 30- Nov. 3, 2008\)](#)  
[The 2nd OCAMI-KNUGRG Joint Differential Geometry Workshop \(Oct. 29- Nov. 1, 2009\)](#)  
[The 3rd KNUGRG-OCAMI Joint Differential Geometry Workshop \(Nov. 4- Nov. 6, 2010\)](#)  
[The 4th KNUGRG-OCAMI Joint Differential Geometry Workshop \(Nov. 2- Nov. 5, 2011\)](#)  
[The 5th KNUGRG-OCAMI Joint Differential Geometry Workshop \(Oct. 31- Nov. 2, 2012\)](#)  
[The 6th OCAMI-KNUGRG Joint Differential Geometry Workshop \(Feb. 1- Feb. 3, 2013\)](#)  
[The 7th KNUGRG-OCAMI Joint Differential Geometry Workshop \(Sep. 30- Oct. 2, 2013\)](#)  
[The 8th OCAMI-KNUGRG Joint Differential Geometry Workshop \(Apr. 14- Apr. 16, 2014\)](#)  
[Poster](#)  
[Kansai Kenshu Center](#)

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**Support** : JSPS Program for Advancing Strategic International Networks to Accelerate the Circulation of  
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(Oct.2014-Mar.2017)

"Mathematical Science of Symmetry, Topology and Moduli, Evolution of International Research Network based on OCAMI"

(Osaka City University - Kobe University - Waseda University, Principal investigator: Yoshihiro Ohnita)

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