## 6. Topology



# Gromov-Hausdorff hyperspaces of $\mathbb{R}^{n}$ 

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2010 Mathematics Subject Classification. 57N20, 57S20, 54H15
Keywords. Gromov-Hausdorff distance, Isometry group, Proper group action, Orbit space, Hilbert cube manifold
M. Gromov first introduced the notion of the Gromov-Hausdorff distance $d_{G H}$ in his ICM 1979 address in Helsinki on synthetic Riemannian geometry. It turns the set $G H$ of all isometry classes of compact metric spaces into a metric space.

Given two compact metric spaces $\left(X, d_{X}\right)$ and $\left(Y, d_{Y}\right)$, let $\mathcal{D}\left(d_{X}, d_{Y}\right)$ denote the set of all possible metrics on the disjoint union $X \cup Y$ that extend the metrics $d_{X}$ and $d_{Y}$. Then $d_{G H}(X, Y)$ is defined to be the infimum of all Hausdorff distances $d_{H}(X, Y)$ for all metrics $d \in \mathcal{D}\left(d_{X}, d_{Y}\right)$. The metric space $\left(G H, d_{G H}\right)$ is called the Gromov-Hausdorff hyperspace. It is a challenging open problem to understand the topological structure of this metric space. This talk contributes towards this problem.

We mainly are interested in the following subspaces of $G H$ denoted by $G H\left(\mathbb{R}^{n}\right), n \geq 1$, and called the Gromov-Hausdorff hyperspace of $\mathbb{R}^{n}$. Here $G H\left(\mathbb{R}^{n}\right)$ is the subspace of $G H$ consisting of the classes $[E] \in G H$ whose representative $E$ is a metric subspace of the Euclidean space $\mathbb{R}^{n}$. One of the results in this talk asserts that $G H$ is homeomorphic to the orbit space $2^{\mathbb{R}^{n}} / E(n)$, where $2^{\mathbb{R}^{n}}$ is the hyperspace of all nonempty compact subsets of $\mathbb{R}^{n}$ endowed with the Hausdorff metric and $E(n)$ is the isometry group of $\mathbb{R}^{n}$. This is applied to prove that $G H$ is homeomorphic to the Hilbert cube with a removed point.

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## A further contribution to properties of remote points in pointfree topology

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2010 Mathematics Subject Classification. 06D22, 54A20, 54E17
Keywords. Frame, remote point, perfect extension, balanced filter, coproduct
Remote points in pointfree topology were introduced in [2], where the study restricted only to points of the Stone-Čech compactification $\beta L$ of a completely regular frame $L$ that are remote from $L$. In this paper we study properties of remote points of extensions of a (completely regular) frame $L$, where, by "an extension of $L$ " is meant a dense onto frame homomorphism $h: M \longrightarrow L$. We draw up characterizations of these remote points, showing the role played
by extensions whose right adjoints preserve disjoint binary joins. We also attempt at determining conditions under which remote points in summands give rise to a remote point in the coproduct of a family of frames.

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## Normal complex surface singularities with rational homology disk smoothings

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2010 Mathematics Subject Classification. 32S30, 14B07, 14J17
Keywords. Surface singularity, Milnor fiber, rational homology disk smoothing
In this talk we show that if the minimal good resolution graph of a normal surface singularity contains at least two nodes (i.e. vertex with valency at least 3 ) then the singularity does not admit a smoothing with Milnor fiber having rational homology equal to the rational homology of the 4 -disk $D^{4}$ (called a rational homology disk smoothing). Combining with earlier results, this theorem then provides a complete classification of resolution graphs of normal surface singularities with a rational homology disk smoothing, verifying a conjecture of J. Wahl regarding such singularities. Indeed, together with a recent result of J. Fowler we get the complete list of normal surface singularities which admit rational homology disk smoothings. This is a joint work with Dongsoo Shin and András Stipsicz.

# Arbitrarily long factorizations in mapping class groups 

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2010 Mathematics Subject Classification. 57N13, 20F38, 57R17, 57M07
Keywords. Mapping class group, Lefschetz fibrations, contact structure

On a compact oriented surface of genus $g$ with $n \geq 1$ boundary components, $\delta_{1}, \delta_{2}, \ldots, \delta_{n}$, we consider positive factorizations of the boundary multitwist $t_{\delta_{1}} t_{\delta_{2}} \cdots t_{\delta_{n}}$, where $t_{\delta_{i}}$ is the positive Dehn twist about the boundary $\delta_{i}$. We prove that for $g \geq 3$, the boundary multitwist $t_{\delta_{1}} t_{\delta_{2}}$ can be written as a product of arbitrarily large number of positive Dehn twists about nonseparating simple closed curves. This fact has immediate corollaries on the Euler characteristics of the Stein fillings of conctact three manifolds.

# Differential $\lambda$ calculus proves Poincare $S_{P L}^{n}$ by quantum induction 

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2010 Mathematics Subject Classification. 56, 81, 19, 79, 51
Keywords. Induction, $\lambda$ calculus, categorical convexity, Lie-Hopf-Hilbert open-categories, simple homotopy type R-torsion, Ricci scalar product

We introduce a differential calculus derived from $\lambda$ calculus in Computer Science to smooth homotopy PL spheres $S^{\prime}$, independantly of dimension. This breakthrough bridges Diff types by PL-Homogeneous categorical induction $H \stackrel{\Gamma \rtimes H}{\mapsto} G / H_{x} \in X_{\Gamma}=\mathcal{R} e p_{G / H}$ of group $H=\cap H_{x}$, also called Spontaneous Symmetry Breaking. The affine crossed representations $\gamma_{x} \in \operatorname{Proj}\left(K_{x}\right) \rtimes H_{x}$ in projective space of $K_{x}=T_{x} \oplus A x, H_{x}$ fixing $x$ but spanning $T_{x}$, are Hopf convex open-categories, fibers of comonoidal double open-category of convex squares $\gamma_{y} \xrightarrow{\gamma_{y / x}^{\prime}} \gamma_{x}$ over charts with coproduct on $X_{\Gamma}$. The commutant comonoidal category $\mathcal{C}^{\prime}$ has full fibers $\mathcal{C}_{x}^{\prime}$ with arrows $\mathfrak{g}_{x}=\Gamma_{x} \rtimes H_{x} \subset \mathfrak{a u t}\left(\operatorname{Proj}\left(K_{x}\right)\right)$, Lie open-category, with 3-dim Reidemeister isotopy counit $\theta=\theta \theta=\theta \oplus \theta \in \mathcal{C}_{\emptyset}^{\prime}=A$ : quantum scalars. The principal action $\mathcal{D}_{x}^{\prime}=\mathcal{C}_{x}^{\prime} \cdot \theta \rightarrow \operatorname{Proj}\left(K_{x}\right)$ is fiber of comonoidal category $\mathcal{D}^{\prime}$, covered by convex open sections. The exponential of $\mathcal{C}^{\prime} \approx \mathcal{D}^{\prime}$ is a $\lambda$ calculus on monoidal category $\mathcal{C}(x \times y, z) \approx \mathcal{D}\left(x .1, z^{y} .1\right)$. It implies a Hilbert $\otimes$ open-category $\mathcal{D}==_{O b} \Gamma^{\prime} \xrightarrow{\tau} A$ with $\Gamma$ invariant Ricci scalar product $\tau, X_{\Gamma}=S p e c \Gamma^{\prime}$. The fundamental triangular relation is $\Gamma^{\prime} \xrightarrow{\lambda}$ $\theta \xrightarrow{\tau} \lambda \xrightarrow{\theta} \Gamma^{\prime}$. The breakthrough $\lambda$-derivative $T_{x} \xrightarrow{\nabla_{T}^{\lambda} a} A$ uses only small enough variations $|T|$, but not arbitrarily small. While the affine changes $\gamma_{y / x}^{\prime}=1$ induct $H=G$ or $e \rightarrow G$, as Quantum groups, the projective changes induct $H \rightarrow G$ via Quantum orbits, by projective
braiding $J \times R \in \operatorname{int}\left(\Gamma^{\prime}\right) \otimes \mathcal{U}_{\mathfrak{h}} \otimes \mathcal{U}_{\mathfrak{h}}$ with modular conjugation $J$. Finite dimensional orbits defined by $\lambda$ Diff-de Rham chain $C\left(\mathcal{D}_{x}\right)$ with boundary-orientation condition have R-torsion $\Gamma_{d}^{\prime} \xrightarrow{\tau^{\Gamma}\left(\text { Det }_{x}\right)} \mathbb{C}, \quad \tau^{\Gamma}(y / x)=\exp \langle x, y\rangle$, determining $G$ by $\tau^{\Gamma}(G)=1$. This construction restricts at each point $x$ of homotopy $n$-sphere $S^{\prime}$ as large cyclic group of rotations $D e t_{x}^{\Gamma}$ on $S^{n} \subset \mathbb{C}^{n+1}$. For $x, y \in S^{\prime}, D e t_{x}^{\Gamma}, D e t_{y}^{\Gamma}$ are PL conjugated, hence $\tau^{\Gamma}(y / x)=1$. By $\tau$-rigidity of cyclic rotations, $x, y$ are $S O_{n+1}$-conjugated, proving $S^{\prime} \approx_{P L} S^{n}$, by quantum induction data $\Gamma^{\prime}=\operatorname{End}\left(\mathbb{C}^{n}\right) \quad S^{\prime}=\Gamma \rtimes S O_{n} \approx_{P L} S^{1} \rtimes S O_{n}=S^{n}=\mathcal{R} e p_{S^{n}}$.

## 00-06-0599

# Isotopy and invariants of Legendrian surfaces 

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2010 Mathematics Subject Classification. 57R17, 57R52, 57R65
Keywords. Legendrian surface, Legendrian isotopy, Legendrian attaching disk surgery, Legendrian yindex

This talk concerns Legendrian isotopy classes of embedded Legendrian surfaces of a contact 5 -manifold $(M, \xi)$. For $(M, \xi)$ being parallelizable we introduce a new numerical invariant, called Legendrian $y$-index, which is defined for any closed orientable Legendrian surface immersed in $(M, \xi)$. This Legendrian $y$-index, a contact analog of the $y$-index constructed in [3], is invariant under Legendrian isotopies and a nontrivial Legendrian isotopy invariant of Legendrian surfaces.

As an example, we construct for each nonnegative integer $g$, an infinite number of smoothly isotopic embedded Legendrian surfaces of genus $g$ in $(M, \xi)$ by way of Legendrian attaching disk surgery, where the surgery is a contact analog of the Lagrangian attaching disk surgery as defined in [3]. For $g$ fixed, we show that these surfaces of genus $g$ have the same classical invariants (Thurston-Bennequin number and rotation class) but different Legendrian $y$-indexes, hence have distinct Legendrian isotopy classes. We remark here that our Legendrian disk surgery coincides with Rizell's Legendrian ambient surgery with $k=1, n=2$ [2].

We will discuss the effect of other types of surgeries on Legendrian $y$-index, including cusp connected sum and stabilization as defined in [1]. We will also comment on the relation between Legendrian $y$-index and other types of Legendrian isotopy invariants.

This is a work in progress.

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v,3.

OO-06-0820

# Torsions of cohomology of real toric varieties 

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2010 Mathematics Subject Classification. 57N65, 57S17, 05E45
Keywords. Real toric manifold, small cover, cohomology ring, odd torsion, nestohedron

We present a formula to compute the rational cohomology ring of a real topological toric manifold, and thus that of a small cover or a real toric manifold, which implies the formula of Suciu and Trevisan. Furthermore, the formula also works for other coefficient $\mathbb{Z}_{q}=\mathbb{Z} / q \mathbb{Z}$, where $q$ is a positive odd integer. As an important application, we construct infinitely many real toric manifolds and small covers whose integral cohomology rings have a $q$-torsion for any positive odd integer $q$.

OO-06-0821

## Volume and topology

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2010 Mathematics Subject Classification. 57M50, 57R19, 53C23
Keywords. Volume, Hyperbolic Manifolds, Bounded Cohomology

We will present some results concerning the behavior of the simplicial volume and the volume of locally symmetric spaces under basic topological operations such as cut and paste. Moreover we will discuss the proportionality principle for simplicial and geometric volume of Riemannian manifolds and our proof (with Sungwoon Kim) of the proportionality principle for noncompact manifolds of pinched negative curvature.

## Unknotting number of some knots

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2010 Mathematics Subject Classification. 57M27, 57M25
Keywords. Unknotting number, Unknotting sequence, Torus knots

We provide a new approach to unknot torus knots and using this new approach, we give an unknotting sequence for every torus knot. A table of knots upto 16 crossings was provided by Hoste and Thistlethwaite, among those, the unknotting numbers of many knots are unknown. By showing most of these knots lying in some unknotting sequence of torus knots, we provide unknotting number for more than 700 knots having crossing numbers between 11 to 16 .

## OO-06-1143

## On Cohen braids

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2010 Mathematics Subject Classification. 57M, 55, 20E99
Keywords. Braid, surface, generating set

Let $M$ be a general connected surface, possibly with boundary components. We denote by $B_{n}(M)$ the $n$-strand braid group on a surface $M$. The operations

$$
d_{i}: B_{n}(M) \rightarrow B_{n-1}(M)
$$

are obtained by forgetting the $i$-th strand of a braid, $1 \leq i \leq n$. We study the system of equations

$$
d_{1} \beta=\cdots=d_{n} \beta=\alpha
$$

where $\alpha$ is a braid in $B_{n-1}(M)$. We obtain that if $M \neq S^{2}$ or $\mathbb{R} P^{2}$ this system of equations has a solution $\beta \in B_{n}(M)$ if and only if

$$
d_{1} \alpha=\ldots=d_{n} \alpha
$$

The set of braids satisfying the last system of equations we call Cohen braids. We also construct a set of generators for the groups of Cohen braids. In the cases of the sphere and the projective plane we give some examples for the small number of strands. This is a joint work with V.G.Bardakov and Jie Wu, see arXiv:0909.3387.

## OO-06-1248

## Further study of kanenobu knots

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2010 Mathematics Subject Classification. 57M27
Keywords. Kanenobu knots, Khovanov homology, crossing number

We determine the rational Khovanov bigraded homology groups of Kanenobu knots. Also, we determine the crossing number for all Kanenobu knots $K(p, q)$ with $p q>0$ or $|p q| \leq$ $\max \{|p|,|q|\}$. In the case where $p q<0$ and $|p q|>\max \{|p|,|q|\}$, we conjecture that the crossing number is $|p|+|q|+8$.

00-06-1347

## Polygonal approximation of knots by quadrisecants

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2010 Mathematics Subject Classification. 57M25
Keywords. Knot, polygonal approximation, quadrisecant, quadrisecant approximation

We consider tame knots in space. Every knot can be deformed to a polygonal knot without changing its knot type. If a set of finitely many points is chosen on a knot, we may straighten each subarc between nearby points of the set to form a polygonal curve. Such a curve is called a polygonal approximation of the given knot. A polygonal approximation of a knot is said to be good if it has the same type as the given knot. A quadrisecant of a knot is a straight line which intersects the knot in four distinct points. Every nontrivial knot can be perturbed to have finitely many quadrisecants. If a knot has finitely many quadrisecants, we may use the secant points to form a polygonal approximation, called the quadrisecant approximation. Quadrisecant approximations are conjectured to be good polygonal approximations. We report on our test of this conjecture on a family of random polygonal unknots.

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OO-06-1393
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## Topology of generalized Bott manifolds

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2010 Mathematics Subject Classification. 57R19, 57R20, 57S25, 14M25
Keywords. Bott manifold, generalized Bott manifold, cohomological rigidity, toric variety, toric topology

A Bott tower (resp., generalized Bott tower) of height $n$ is a sequence of projective bundles

$$
B_{\bullet}: B_{n} \xrightarrow{\pi_{n}} B_{n-1} \xrightarrow{\pi_{n-1}} \cdots \xrightarrow{\pi_{2}} B_{1} \xrightarrow{\pi_{1}} B_{0}=\{\text { a point }\},
$$

where each $\pi_{i}$ is the projectivization of a Whitney sum of two (resp., finitely many) complex line bundles. We call $B_{n}$ an $n$-stage Bott manifold (resp. generalized Bott manifold). A one-stage Bott manifold is a complex projective space, and a two-stage Bott manifold is known as a Hirzebruch surface. Due to Hierzebruch, it is known that topological (smooth) type of Hirzebruch surfaces is completely determined by its cohomology rings. So it leads to conjecture that two generalized Bott manifolds are diffeomorphic or not if their cohomology rings are isomorphic as graded rings. It is now called the Cohomologically rigidity conjecture for generalized Bott manifolds. This conjecture is still open, but there are some partial results. In this talk, we survey the current progress on the conjecture, and we shall present many affirmative evidences of that any cohomology ring isomorphism between two generalized Bott manifolds is indeed realizable by a certain diffeomorphism. Furthermore, we also discuss about the smooth classification of toric varieties.

## OO-06-1532

## Simplicial volume of noncompact manifolds

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2010 Mathematics Subject Classification. 53C23, 53C35
Keywords. Simplicial volume, locally symmetric spaces, bounded cohomology

We report recent advances on the study of the simplicial volume of noncompact manifolds, including Q-rank 1 locally symmetric spaces. If time permits we will discuss some applications and related open problems.

## OO-06-1585

## Ideal coset invariants for surface-links in 4-space

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2010 Mathematics Subject Classification. 57Q45, 57M25, 57M27
Keywords. Marked graph diagram, surface-link, invariant of surface-link, Gröbner basis, Yoshikawa move

A surface-link or a knotted surface $L$ of $n$ components ( $n \geq 1$ ) is $n$ mutually disjoint connected and closed (possibly orientable or non-orientable) 2-manifolds smoothly (or piecewise linearly and locally flatly) embedded in the oriented 4 -space. In 2009, Lee defined a polynomial $[[D]]$ for marked graph diagrams $D$ of surface-links in 4 -space by using a state-sum model involving a given classical link invariant. In this talk, I would like to discuss some obstructions to obtain an invariant for surface-links represented by marked graph diagrams $D$ by using the polynomial $[[D]]$ and introduce an ideal coset invariant for surface-links, which
is defined to be the coset of the polynomial $[[D]]$ in a quotient ring of a certain polynomial ring modulo some ideal and represented by a unique normal form, i.e. a unique representative for the coset of $[[D]]$ that can be calculated from $[[D]]$ with the help of a Gröbner basis package on computer.

# Exact computation and the cusped hyperbolic census 

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2010 Mathematics Subject Classification. 57N10, 57-04, 57Q15, 57N16
Keywords. 3-manifolds, hyperbolic manifolds, exact computation, census, mathematical software

Over its quarter-century history, the "SnapPea census" of cusped finite-volume hyperbolic 3 -manifolds has been an invaluable resource for low-dimensional topologists. In its modern form it contains 21,918 cusped 3-manifolds, believed to represent all cusped finite-volume hyperbolic 3-manifolds that can be built from $\leq 8$ ideal tetrahedra.

Despite its long history, however, questions of accuracy remain unresolved. The key issues are (i) that the manifolds in the census are only those for which the software SnapPea identified a geometric triangulation; and (ii) that SnapPea uses inexact floating point arithmetic to test whether a triangulation is geometric. In recent work, numerical techniques of Moser and Hoffman et al. have been able to show that false positives do not occur. However, false negatives still remain a possibility: a hyperbolic manifold could be omitted from the census because a geometric triangulation was incorrectly thought to be non-geometric (due to roundoff error), or because it might have no geometric triangulation at all.

Here we finally resolve these issues, and give a rigorous proof that the SnapPea census is correct. The proof is computationally intensive and algorithmically non-trivial: in essence we exhaustively enumerate all candidate ideal triangulations (roughly 230 million in total), and for each triangulation we rigorously certify that it is either (i) homeomorphic to a census manifold, or (ii) non-hyperbolic. Unlike the numerical techniques of Moser and Hoffman et al., here we work with discrete techniques such as normal surface theory, Dehn fillings, and combinatorial manipulation and analysis of triangulations.

The broader implication of this work is to highlight the increasing feasibility of exact computation on a massive scale for problems surrounding hyperbolic 3-manifolds, complementing the significant body of inexact but highly efficient numerical software (such as SnapPea) that is already in use.

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OO-06-1624
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## Cobordisms of Lefschetz fibrations on 4-manifolds

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2010 Mathematics Subject Classification. 55R55, 57R90, 57N13
Keywords. Lefschetz fibration, cobordism, mapping class group

Topological Lefschetz fibrations are defined so far to be maps over a surface with only complex non-degenerate singularities. We propose a natural generalization of this notion, by allowing the base manifold to have arbitrary dimension. We then define the cobordism groups of Lefschetz fibrations, along the lines of singular bordism theory. We also attempt to compute these groups in low dimensions, by means of universal Lefschetz fibrations, and we illustrate their relationship with the singular bordism groups of certain manifolds that can be constructed in a rather explicit way.

## OO-06-1631

## On the Alexander biquandles for oriented surface-links via marked graph diagrams

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2010 Mathematics Subject Classification. 57Q45, 57M25
Keywords. Alexander biquandle, marked graph diagram, surface-link, invariant of surface-link, invertible surface-link

By a surface link, or knotted surface, $\mathcal{L}$ of $n$ components ( $n \geq 1$ ) we mean $n$ mutually disjoint connected and closed (possibly orientable or nonorientable) 2-manifolds $F_{1}, \ldots, F_{n}$ smoothly (or piecewise linearly and locally flatly) embedded in the standardly oriented 4space $\mathbb{R}^{4}$ (or $S^{4}$ ). In the case when each component $F_{i}$ is oriented, $\mathcal{L}$ is called an oriented surface link. In 2009, T. Carrell defined the fundamental biquandle of an oriented surfacelink by a presentation obtained from its broken surface diagram, which is an invariant up to isomorphism of the surface-link. Recently, S. Ashihara gave a method to calculate the fundamental biquandle of an oriented surface-link from its marked graph diagram (ch-diagram). In this talk I would like to discuss the fundamental Alexander biquandles of oriented surfacelinks via marked graph diagrams, derived computable invariants and their applications to detect non-invertible oriented surface-links.

## Semi-separation axiom of digital topological spaces

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2010 Mathematics Subject Classification. 54C10, 54C08
Keywords. Digital topology, semi-separation axiom, Alexandroff space, Khalimsky topology, locally finite topology

The paper proves that each subspace of an Alexandroff $T_{0}$-space is semi- $T_{\frac{1}{2}}$. In particular, any subspace of the folder $X^{n}$, where $n$ is a positive integer and $X$ is either the Khalimsky line $\left(\mathbb{Z}, \tau_{K}\right)$, the Marcus-Wyse plane $\left(\mathbb{Z}^{2}, \tau_{M W}\right)$ or any partially ordered set with the upper topology is semi- $T_{\frac{1}{2}}$. Then we study the basic properties of spaces possessing the axiom semi$T_{\frac{1}{2}}$ such as finite productiveness and monotonicity. Besides, we investigate some properties of the semi-separtion axiom of Khalimsky topological spaces. Finally, this talk suggests some applications of this approach.

OO-06-2055

## The fixed point property everywhere

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2010 Mathematics Subject Classification. 55M20, 55U10, 54H25, 06A07
Keywords. Fixed point property, Weak homotopy types, Lefschetz fixed point theorem, Finite topological spaces

If $K$ is a polyhedron homotopy equivalent to the 1 -dimensional sphere, then there exists a fixed point free map $f: K \rightarrow K$. However, we will show that there exists a (non-Hausdorff) space $X$ weak homotopy equivalent to $S^{1}$ with the fixed point property. Moreover, the latter claim is true if we replace $S^{1}$ by any connected compact $C W$-complex.

00-06-2079

## Discretization of topological and quantum spaces

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2010 Mathematics Subject Classification. 54A10, 46L85
Keywords. Discretization, compactification, topological space, quantum space, quantum group

There are several compactification procedures in topology, but there is only one standard discretization, namely replacing the original topology with a discrete topology. We give a notion of discretization which is dual (in categorical sense) to compactification and give examples of discretizations. We study discretizations of (abelian) groups, especially those constructed by dualizing a compactification. We also introduce a notion of discretization for quantum spaces (non commutative topologies) and briefly study discretizations of locally compact quantum groups.

# A topologically minimal, weakly reducible, unstabilized Heegaard splitting of genus three is critical 

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2010 Mathematics Subject Classification. 57M50
Keywords. Topologically minimal surface, Heegaard splitting, topological index, 3-manifold

Let $(\mathcal{V}, \mathcal{W} ; F)$ be a weakly reducible, unstabilized, genus three Heegaard splitting in an orientable, irreducible 3 -manifold $M$. In this talk, we prove that if every weak reducing pair of $F$ gives the same generalized Heegaard splitting after weak reduction up to isotopy, then the disk complex $\mathcal{D}(F)$ is contractible. Indeed, we also prove that $F$ is critical otherwise. Hence, the topological index of $F$ is two if $F$ is topologically minimal.

## 00-06-2309

## Symplectic and nonsymplectic 6-manifolds

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Keywords. 6-manifolds, symplectic

A topological construction of simply connected smooth six manifolds with $w_{2}=0, b_{2}=1$ and $b_{3}=0$ will be explained. This construction may result with symplectic manifolds as well as nonsymplectic manifolds. The reason for unsimilar outcomes will be explored.

## OO-06-2336

## On span of incomplete real flag manifolds

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2010 Mathematics Subject Classification. 57R25, 57R20, 55R10
Keywords. Real flag manifold, span, vector field problem, Stiefel-Whitney classes, fiber bundles

In this paper, we give bounds for the span of certain infinite families of the incomplete flag manifolds using suitable fiberings where the flag manifold is either a total space or base space.

We also use non-vanishing Stiefel- Whitney classes to obtain exact values of the span for some of the manifolds.

## OO-06-2343

## Sol $_{1}^{4}$-geometry

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2010 Mathematics Subject Classification. 20H15, 22E25, 20F16, 57S25
Keywords. Solvmanifolds, Infra-solvmanifolds, Bieberbach theorems, crystallographic groups

We classify all compact manifolds modeled on the 4-dimensional solvable Lie group $\mathrm{Sol}_{1}{ }^{4}$. The maximal compact subgroup of $\operatorname{Isom}\left(\mathrm{Sol}_{1}^{4}\right)$ is $D_{4}=\mathbb{Z}_{4} \rtimes \mathbb{Z}_{2}$. We shall exhibit an infrasolvmanifold with $\mathrm{Sol}_{1}{ }^{4}$-geometry whose holonomy is $D_{4}$. This implies that all possible holonomy groups do occur; $\{1\}$, $\mathbb{Z}_{2}$ ( 5 families), $\mathbb{Z}_{4}, \mathbb{Z}_{2} \times \mathbb{Z}_{2}$ ( 5 families), and $\mathbb{Z}_{4} \rtimes \mathbb{Z}_{2}$ (2 families). Of course, this includes the classification of 3-dimensional infra-Sol manifolds. We also show that all infra-Sol ${ }_{1}{ }^{4}$-manifolds are un-oriented boundaries.

# The Nielsen and Reidemeister numbers of maps on infra-solvmanifolds of type (R) 

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2010 Mathematics Subject Classification. 37C25, 58F20
Keywords. Infra-solvmanifold, Nielsen number, Reidemeister number, Nielsen zeta function, Reidemeister zeta function

We prove the rationality, the functional equations and calculate the radii of convergence of the Nielsen and the Reidemeister zeta functions of continuous maps on infra-solvmanifolds of type (R). We find a connection between the Reidemeister and Nielsen zeta functions and the Reidemeister torsions of the corresponding mapping tori. We show that if the Reidemeister zeta function is defined for a homeomorphism on an infra-solvmanifold of type (R), then this manifold is an infra-nilmanifold. We also prove that a map on an infra-solvmanifold of type (R) induced by an affine map minimizes the topological entropy in its homotopy class and it has a rational Artin-Mazur zeta function. Finally we prove the Gauss congruences for the Reidemeister and Nielsen numbers of any map on an infra-solvmanifolds of type (R) whenever all the Reidemeister numbers of iterates of the map are finite. Our main technical tool is the averaging formulas for the Lefschetz, the Nielsen and the Reidemeister numbers on infra-solvmanifolds of type (R).

## OP-06-0351

# A new generalization of the Khovanov homology 

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2010 Mathematics Subject Classification. 57M25, 57M27
Keywords. Knots, Khovanov Homology, Anyonic braiding, knot invariants

In this talk, we give a new generalization of the Khovanov homology. The construction begins with a Frobenius-algebra-like object in a category of graded vector-spaces with an anyonic braiding, with most of the relations weaken to hold only up to phase. The construction of Khovanov can be adapted to give a new link homology theory from such data. Both Khovanov's original theory and the odd Khovanov homology of Ozsvath, Rasmussen and Szabo arise from special cases of the construction in which the braiding is a symmetry.

## OP-06-0384

## Groups of homeomorphisms and diffeomorphisms of non-compact manifolds with the Whitney topology

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2010 Mathematics Subject Classification. 57S05, 58D05, 57N20, 46A13
Keywords. Groups of homeomorphisms and diffeomorphisms, Whitney topology, Non-compact manifolds, LF spaces, Box products

We study topological properties of groups of homeomorphisms and diffeomorphisms of noncompact manifolds with the Whitney topology. Suppose $M$ is a non-compact connected $n$ manifold. The group $\mathcal{H}(M)$ of homeomorphisms of $M$ endowed with the Whitney topology is a topological group. The subgroup $\mathcal{H}_{c}(M)$ of compactly supported homeomorphisms of $M$ is paracompact and locally contractible. The connected component $\mathcal{H}(M)_{0}$ of $i d_{M}$ in $\mathcal{H}(M)$ is an open normal subgroup of $\mathcal{H}_{c}(M)$. The pair $\left(\mathcal{H}(\mathbb{R}), \mathcal{H}_{c}(\mathbb{R})\right)$ is homeomorphic to the pair $\left(\square^{\omega} l_{2}, \square^{\omega} l_{2}\right)$ of countable box and small box products of $l_{2}$. The topological classification of LF spaces implies that $\square^{\omega} l_{2} \approx l_{2} \times \mathbb{R}^{\infty}$, where $\mathbb{R}^{\infty}$ is the direct limit of Euclidean spaces $\mathbb{R}^{n}$. In the case where $n=2$, (i) the pair $\left(\mathcal{H}(M), \mathcal{H}_{c}(M)\right)$ is locally homeomorphic to the pair $\left(\square^{\omega} l_{2}, \square^{\omega} l_{2}\right)$ at $i d_{M}$, (ii) $\mathcal{H}_{c}(M)$ is a topological $\left(l_{2} \times \mathbb{R}^{\infty}\right)$ manifold and (iii) $\mathcal{H}(M)_{0} \approx l_{2} \times \mathbb{R}^{\infty}$. When $M$ is a smooth non-compact connected $n$ manifold, we can consider the group $\mathcal{D}(M)$ of diffeomorphisms of $M$ and its subgroups $\mathcal{D}_{c}(M)$ and $\mathcal{D}(M)_{0}$, endowed with the Whitney $C^{\infty}$-topology. For any dimension $n$ the pair
$\left(\mathcal{D}(M), \mathcal{D}_{c}(M)\right)$ is locally homeomorphic to the pair $\left(\square^{\omega} l_{2}, \square^{\omega} l_{2}\right)$ at $i d_{M}$ and $\mathcal{D}_{c}(M)$ is a topological $\left(l_{2} \times \mathbb{R}^{\infty}\right)$-manifold. The pair $\left(\mathcal{D}(\mathbb{R}), \mathcal{D}_{c}(\mathbb{R})\right)$ is homeomorphic to the pair $\left(\square^{\omega} l_{2}, \square^{\omega} l_{2}\right.$ ). For $n=2,3$, we have also obtained some results on topological types of $\mathcal{D}(M)_{0}$.

## References

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# $S_{5}$ action with rank 1 isotropy on a $G \mathbf{C W}$-complex $\mathbf{X}$ homotopy equivalent to a sphere 

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2010 Mathematics Subject Classification. 57S17, 18Gxx, 20J05, 55U15, 57S25
Keywords. Group actions, orbit category, equivariant CW-complex, isotropy subgroup

A good algebraic setting for studying actions of a group with isotropy given in a given family of subgroups is provided by the modules over the orbit category. In this talk, I will briefly talk about the orbit category and then I will give the construction of the finite $G$ - CW complex homotopy equivalent to a sphere on which the group $S_{5}$ acts with isotropy in the family of cyclic subgroups.

This talk is a part of the paper Equivariant CW Complexes and The Orbit Category which is a joint work with I. Hambleton and E. Yalcin.

## OP-06-0535

## Complete classifications of exceptional surgeries on Montesinos knots and alternating knots

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Keywords. Montesinos knot, alternating knot, exceptional surgery, Seifert fibered surgery

In the talk, we will consider exceptional surgeries on two family of knots in the 3-sphere; Montesinos knots and alternating knots. Actually we will give complete classifications of exceptional surgeries on such knots. For the case of Montesinos knots, based on joint works by the first two authors, the key ingredient is the use of Rassmussen invariant. The final step for the case of Montesinos knots and the case of alternating knots are achieved by (super-)computer-aided calculations by the first and the last authors.

## OP-06-0637

## Ropelength criticality

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2010 Mathematics Subject Classification. 57M25, 49J52, 53A04
Keywords. ropelength, tight knot, simple clasp, constrained minimization

The ropelength problem asks for the minimum-length configuration of a knotted diameterone tube embedded in Euclidean three-space. The core curve of such a tube is called a tight knot, and its length is a knot invariant measuring complexity. In terms of the core curve, the thickness constraint has two parts: an upper bound on curvature and a self-contact condition.

In work with Cantarella, Kusner and Fu, we give a set of necessary and sufficient conditions for criticality with respect to this constraint, based on a new version of the Kuhn-Tucker theorem. The key technical difficulty is to compute the derivative of thickness under a smooth perturbation. This is accomplished by writing thickness as the minimum of a $C^{1}$-compact family of smooth functions in order to apply a theorem of Clarke. We give a number of applications, including a classification of the supercoiled helices formed by critical curves with no self-contacts (constrained by curvature alone) and an explicit but surprisingly complicated description of the clasp junctions formed when one rope is pulled tight over another.

## OP-06-0709

## Reidemeitser torsion of a homology 3-sphere surgeried along the ( $\mathbf{p}, \mathbf{q}$ )-torus knot for $S L(2 ; \mathbb{C})$-represenatations

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2010 Mathematics Subject Classification. 57M27, 57Q10
Keywords. Reidemeitser torsion, SL(2,C)-represenation, homology 3-sphere
Reidemeister torsion is a classical invariant introduced to classify lens spaces up to PLisomorphism. It is defined for a pair of a PL-manifold and a linear representation of the fun-
damental group satisfying the acyclicity condition. The acyclic condition is that all twisted homology are vanishing. In the study of low dimensional topology, it has an important role with relations to Alexander polynomial, twisted Alexander polynomial, quantum invariants, and hyperbolic volume.

In this talk we consider this invariant for a closed 3-manifold with an $S L(2 ; \mathbb{C})$-representation. Let $M$ be a 3-manifold and $\rho: \pi_{1}(M) \rightarrow S L(2 ; \mathbb{C})$ a representation. By considering $\mathbb{C}^{2}$-coefficients twisted homology, we can define Reidemeister torsion $\tau_{\rho}(M)$ which depends on an acyclic representation $\rho$. Namely $\tau_{\rho(M)}$ is a function on the space of $S L(2 ; \mathbb{C})$ representations. Here we consider the set of all values $\left\{\tau_{\rho}(M)\right\}$ and the polynomial $P_{M}(t)=$ $\Pi\left(t-\tau_{\rho}(M)\right)$. This is a polynomial invariant of $M$ introduced by Dennis Johnson. In the case of the homology 3 -sphere $\Sigma(2,3,6 n \pm 1)$ obtained by the $1 / n$-Dehn surgery along the trefoile knot $T(3,2)$, D. Johnson gave the recursive formula for this polynomial $P_{M}(t)=$ $\Pi\left(t-\tau_{\rho}(M)\right)$ in his famous unpublished lecture notes; A geometric form of Casson's invariant and its connection to Reidemeister torsion. Here we consider a homology 3-sphere $\Sigma(p, q, p q n \pm 1)$ obtained by the $1 / n$-Dehn surgery along ( $\mathrm{p}, \mathrm{q}$ )-torus knot. By applying Johnson's arguments to this manifold, we can give an explicit formula of $P_{\Sigma(p, q, p q n \pm 1)}(t)$ by using Tchebyshev polynomials on $\cos \theta$.

OP-06-0724

## Prime decompositions of topological objects

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## 2010 Mathematics Subject Classification.

Keywords. 3-manifolds, knotted graphs, 3-orbifolds, virtual knots, Diamond Lemma

We describe a far generalization of the famous Diamond Lemma, which had been published by Newman in 1942 and turned to be very useful in algebra and functional analysis. We replace his confluence condition by so-called mediator condition, which has a clear topological meaning. Using this new Diamond Lemma, we get several interesting results:

1. The Kneser-Milnor prime decomposition theorem (new proof).
2. The Swarup theorem for boundary connected sums for orientable 3-manifolds (new proof and generalization two non-orientable case).
3. A spherical splitting theorem for knotted graphs in 3-manifolds.
4. Counterexamples to the prime decomposition theorem for 3-orbifolds. During a long time the uniqueness of prime decompositions of 3 -orbifolds had been accepted by mathematical community as a folklore theorem. So the existence of counterexamples is quite unexpected.
5. A new theorem on annular splittings of 3-manifolds, which is independent of the JSJdecomposition theorem.
6. Prime decomposition theorem for virtual knots (new result).

## OP-06-0744

# Classification of transitive Lie algebroids from categorical point of view 

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2010 Mathematics Subject Classification. 55R, 57R, 58H
Keywords. Transitive Lie algebroid, coupling, homotopy classification

Transitive Lie algebroids have specific properties that allow to look at the transitive Lie algebroid as an element of the object of a homotopy functor. Each transitive Lie algebroids can be described as a vector bundle endowed with additional structures that admits a construction of inverse image generated by a smooth mapping of smooth manifolds.

Due to K.Mackenzie (General Theory of Lie Groupoids and Lie Algebroids, 2005) the construction can be managed as a homotopy functor $T L A_{\mathfrak{g}}$ from category of smooth manifolds to the transitive Lie algebroids. Hence one can construct a classifying space $\mathcal{B}_{\mathfrak{g}}$ such that the family of all transitive Lie algebroids with fixed Lie algebra $\mathfrak{g}$ over the manifold $M$ has one-to-one correspondence with the family of homotopy classes of continuous maps $\left[M, \mathcal{B}_{\mathfrak{g}}\right]: T L A_{\mathfrak{g}}(M) \approx\left[M, \mathcal{B}_{\mathfrak{g}}\right]$.

The description of the classifying space $\mathcal{B}_{\mathfrak{g}}$ is reduced to classification of coupling between Lie algebra bundle (LAB) and the tangent bundle. We have defined a new topology on the group $\operatorname{Aut}(\mathfrak{g})$ of all automorphisms of the Lie algebra $\mathfrak{g}$, say $\operatorname{Aut}(\mathfrak{g})^{\delta}$, and prove that there is a one-to-one correspondence between the family $\operatorname{Coup}(L)$ of all coupling of the Lie algebra bundle $L$ with fixed finite dimensional Lie algebra $\mathfrak{g}$ as the fiber and the structural group $\operatorname{Aut}(\mathfrak{g})$ of all automorphisms of Lie algebra $\mathfrak{g}$ and the tangent bundle $T M$ and the family $L A B^{\delta}(L)$ of equivalent classes of local trivial structures with structural group $\operatorname{Aut}(\mathfrak{g})$ endowed with new topology $\operatorname{Aut}(\mathfrak{g})^{\delta}$.

OP-06-0749

## Mackenzie obstruction for existing of transitive Lie algebroid

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2010 Mathematics Subject Classification. 55R, 57R, 58H
Keywords. Transitive Lie algebroid, coupling, homotopy classification, Mackenzie obstruction

Let $\mathfrak{g}$ be a finite dimensional Lie algebra and $L$ be a Lie algebra bundle (LAB). Given a coupling $\Xi$ between LAB $L$ and tangent bundle $T M$ of the manifold $M$ generates so called the Mackenzie obstruction $\operatorname{Obs}(\Xi) \in H^{3}(M ; Z L)$ for existing of transitive Lie algebroid (K.Mackenzie, General Theory of Lie Groupoids and Lie Algebroids, 2005, p.279).

We present two case of calculating of the Mackenzie obstruction. In the case of commutative algebra $\mathfrak{g}$ the group $\operatorname{Aut}(\mathfrak{g})^{\delta}$ is isomorphic to the group of all matrices $G L(\mathfrak{g})$ with discrete topology. In this case, the coupling $\Xi$ coincides with a flat connection $\nabla$ in a flat bundle $L$, i.e. $R^{\nabla} \equiv 0$. This means that the form $\Omega$ can be chosen trivial, i.e. $d^{\nabla} \Omega=0$. So the obstacle for coupling of $\operatorname{Obs}(\Xi)$ equals to zero.

The second case describe the Mackenzie obstruction for simply connected manifolds. We prove that for simply connected manifolds $\operatorname{Obs}(\Xi)=0 \in H^{3}\left(M ; Z L ; \nabla^{Z}\right)$

## OP-06-0790

## Samelson products in function spaces

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2010 Mathematics Subject Classification. 55P62, 55Q15
Keywords. Lie model, Lie algebra of derivations, Samelson product

We study Samelson products on models of function spaces. Given a map $f: X \longrightarrow Y$ between 1 -connected spaces and its Quillen model $\mathbb{L}(f): \mathbb{L}(V) \longrightarrow \mathbb{L}(W)$, there is an isomorphism of graded vector spaces $\Theta: H_{*}\left(\operatorname{Hom}_{T V}(T V \otimes(\mathbb{Q} \oplus s V), \mathbb{L}(W))\right) \longrightarrow$ $H_{*}(\mathbb{L}(W) \oplus \operatorname{Der}(\mathbb{L}(V), \mathbb{L}(W)))$. We define a Samelson product on $H_{*}\left(\operatorname{Hom}_{T V}(T V \otimes\right.$ $(\mathbb{Q} \oplus s V), \mathbb{L}(W)))$.

## OP-06-0812

## An unknotting operation using polynomial representation of long knots

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2010 Mathematics Subject Classification. 57M25
Keywords. Unknotting Operation, Long Knots or Open Knots

For every knot-type $K\left(\mathbb{R} \hookrightarrow \mathbb{R}^{3}\right)$, there exist real polynomials $f(t), g(t)$ and $h(t)$ such that the map $t \mapsto(f(t), g(t), h(t))$ from $\mathbb{R}$ to $\mathbb{R}^{3}$ represents $K$ and in fact this map defines an embedding of $\mathbb{C}$ in $\mathbb{C}^{3}$. In this presentation, we show that changing one of the polynomial in the polynomial representation of knot-type $K$, provides an unknotting operation. In particular,
we show that by continuously deforming the polynomial which provides the under/over crossing information for the knot-type $K$ in a particular way, it is possible to change a crossing from over (under) to under (over).

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OP-06-0946
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# A topological proof of a version of Artin's induction theorem 

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2010 Mathematics Subject Classification. 55M99, 20C99
Keywords. Euler characteristic, Representations, representation ring, Artin's induction theorem

We define an Euler characteristic $\chi(X, G)$, for a finite cell complex $X$ with a finite group $G$ acting cellularly on it. Then, each $K_{i}(X)$ (a complex vector space with basis the $i$-cells of $X$ ) is a representation of $G$, and we define the $\chi(X, G)$ to be the alternating sum of the representations $K_{i}(X)$, as an element of the representation ring $R(G)$ of $G$. By adapting the ordinary proof that

$$
\sum_{i}(-1)^{i} \operatorname{dim}_{\mathbb{C}} K_{i}(X ; \mathbb{C})=\sum_{i}(-1)^{i} \operatorname{dim}_{\mathbb{C}} H_{i}(X ; \mathbb{C})
$$

we prove that there is another definition of $\chi(X, G)$ with the alternating sum of the representations $H_{i}(X)$, again as elements of the representation ring $R(G)$ of $G$. We also give a formula for the character of $\chi(X, G)$ in terms of the ordinary Euler characteristic of the fixed point set $X^{g}$. Finally, we prove a weaker version of Artin's induction theorem, stating that if $G$ is a group with an irreducible representation of dimension greater than 1 , then every character of $G$, is a rational linear combination of characters induced up from Abelian subgroups.

## OP-06-1080

## The symmetric squares of quaternionic projective space

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2010 Mathematics Subject Classification. 57R18
Keywords. Orbifold, Symmetric Square, Quaternionic projective space, Integral cohomology ring
This talk concerns the symmetric square $X$ of a quaternionic projective space; by definition, $X$ underlies the global quotient orbifold associated to the involution that interchanges the factors of the cartesian square. I shall describe the geometry of $X$ in terms of the braid space and the projectivisation of the tangent bundle of the projective space. This leads to
a calculation of the integral cohomology ring of $X$, whose product structure is somewhat delicate. The mod 2 cohomology ring and the action of the Steenrod algebra follow rather more straightforwardly. I shall explain comparisons with the integral and mod 2 equivariant cohomology of the global quotient, which are easier to compute but which illuminate and assist the original calculation. Throughout the talk I shall refer to the example of projective 3 -space, for which $X$ has dimension 24 ; this is sufficiently simple to describe in some detail, but difficult enough to be representative of the general case. My talk describes work that I expect to form part of my PhD thesis in 2015.

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OP-06-1096
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# Buchstaber Invariant - generalized chromatic number of simplicial complexes 

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2010 Mathematics Subject Classification. 58D19, 57.01, 52B05
Keywords. Torus action, Buchstaber invariant, chromatic number, matroids, simple polytopes

Buchstaber invariant - combinatorial invariant of simple polytopes and simplicial complexes arising from toric topology. With each simplicial $(n-1)$-dimensional complex $K$ on $m$ vertices we can associate a topological space - $(m+n)$-dimensional moment-angle complex $\mathcal{Z}_{K}$ with a canonical action of a compact torus $T^{m}$. The topology of $\mathcal{Z}_{K}$ and of the action depends only on the combinatorics of $K$, which gives a tool to study combinatorics of polytopes and simplicial complexes in terms of topology of $\mathcal{Z}_{K}$ and the action and vice versa. Then $s(K)$ is equal to the maximal dimension of torus subgroups $H \subset T^{m}, H \simeq T^{k}$ that act freely on $\mathcal{Z}_{K}$.

The Buchstaber invariant plays important role in toric topology, since it captures the information wheather a simple polytope admits at least one quasi-toric manifold, but it is also interesting as a new characteristic of polytopes and complexes. It has been studied since 2001 by I. Izmestiev, M. Masuda and Y. Fukukawa, A. Ayzenberg, the author, and some others. It turned out that it is connected to many modern and classical areas of mathematics: 1) it can be considered as a generalization of a classical chromatic number of a graph to simplicial complex: $m-s(K)$ is equal to the smallest rank of free abelian group such that there exists a coloring of vertices of $K$ with elements corresponding to a simplex being a part of some basis; 2$) s(K)$ is equal to the maximal number $k$, such that there exists a rational point on grassmanian $G_{k}\left(\mathbb{R}^{m}\right)$ satisfying certain system of equations on Plucker coordinates; 3)ana$\log$ of $s(K)$ for the $\mathbb{Z}_{2}$-action on small cover is closely related to the theory of binary matroids and the results of this theory recently gave new view on Buchstaber invariant. We are going to present results and problems of the Buchstaber number theory.

# On the concept of bornology in the context of many-valued mathematicl structures 

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2010 Mathematics Subject Classification. 54A40, 46A17
Keywords. Fuzzy set, Bornology, Fuzzy metric, Fuzzy topogy, Many-valued mathematical structures

In order to apply the conception of boundedness to the case of general topological spaces Hu Sze-Tsen introduced the notion of a bornology (S.-T. Hu, Boundedness in a topological space, J. Math. Pures Appl., 78 (1949), 287-320.) Actually a bornology on a set $X$ is an ideal of its subsets containing all singletons.Given bornological spaces $\left(X, \mathcal{B}_{X}\right),\left(Y, \mathcal{B}_{Y}\right)$ a mapping $f:\left(X, \mathcal{B}_{X}\right) \rightarrow\left(Y, \mathcal{B}_{Y}\right)$ is bounded if the image $f(A)$ of every set $A \in \mathcal{B}_{X}$ belongs to $\mathcal{B}_{Y}$. Important examples of bornological spaces are: a topological space and its relatively compact sets; a metric space and its bounded subsets; a uniform space and its totally-bounded subsets.

We introduce the concept of a many-valued $L$-fuzzy bornology, or an ( $L, M$ )-bornology for short, where $\left(L, \wedge_{L}, \vee_{L}\right)$ is a complete lattice and ( $M, \wedge_{M}, \vee_{M}, *$ ) is a cl-monoid (G. Birkhoff, Lattice Theory, AMS Providence, RI, 1995).Namely, an ( $L, M$ )-bornology on a set $X$ is a mapping $\mathcal{B}: L^{X} \rightarrow M$, such that:
(1) $\mathcal{B}(\{x\})=1_{M} \quad \forall x \in X\left(1_{M}\right.$ is the top element of $\left.M\right)$;
(2) $U \subseteq V \Longrightarrow \mathcal{B}(U) \geq \mathcal{B}(V) \forall U, V \in L^{X}$;
(3) $\mathcal{B}(U \cup V) \geq \mathcal{B}(U) * \mathcal{B}(V) \forall U, V \in L^{X}$.

A mapping $f:\left(X, \mathcal{B}_{X}\right) \rightarrow\left(Y, \mathcal{B}_{Y}\right)$ is called bounded if $\mathcal{B}_{Y}(f(A)) \geq \mathcal{B}_{X}(A) \forall A \in L^{X}$.
Some results and problems concerning the category of $(L, M)$-bornological spaces will be discussed. $(L, M)$-bornologies generated by statistical metrics will be constructed and ( $L, M$ )-bornologies reflecting compactness-type properties in fuzzy topological spaces will be described.

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## OP-06-1132

## Stunted weighted projective spaces and orbifold Thom modules

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2010 Mathematics Subject Classification. 57R18
Keywords. Weighted projective space, Orbibundle, Stunted projective space, Thom isomorphism

This talk describes joint work with Alastair Darby and Beverley O’Neill. We introduce stunted weighted projective spaces $W=W_{k}^{n}$, and present some of their topological properties. By definition, each $W$ is a quotient space of an $n$ - complex dimensional weighted projective space $P$; it is obtained by collapsing the subspace corresponding to a $k$-element subset of the weights of $P$, for some $0 \leq k \leq n+1$. We use classic work of Kawasaki to compute the integral cohomology ring of $W$ (which generally involves non-trivial torsion), and outline its relationship with certain minimal cellular realisations. By mimicking the unweighted case, it is possible to identify each $W$ as the Thom space of an orbibundle over a lower dimensional weighted projective space $P^{\prime}$. We may then interpret the integral cohomology ring of $W$ as a module over that of $P^{\prime}$, whose structure is determined by the weights. When every weight is 1 the module becomes cyclic, and we recover the well-known Thom isomorphism. The 3 -dimensional case with weights $1,2,3$, and 4 provides a valuable motivating example, whose various stunted quotients illustrate many of the phenomena that arise when dealing with arbitrary weights.

# Meridional and non-meridional epimorphisms between knot groups 

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2010 Mathematics Subject Classification. 57M25
Keywords. Knot group, epimorphism, meridian

We will consider epimorphisms between knot groups. Especially, we will focus on the image of a meridian under such an epimorphism. A homomorphism between knot groups is called meridional if it preserves their meridians. The existence of a meridional epimorphism introduces a partial order on the set of prime knots. We will determine the pairs of prime knots with up to 11 crossings which admit meridional epimorphisms between their knot groups. Moreover, we will describe some examples of non-meridional epimorphisms explicitly.

## OP-06-1219

## $\mathbb{Z}_{2}$-actions on the 3-Sphere and 2-Torus.

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2010 Mathematics Subject Classification. 57M12, 57N12, 55M25
Keywords. Triangulations of 2-manifolds, Group actions on 2-manifolds, quotient maps and quotient spaces

We discuss various, non-equivalent, $\mathbb{Z}_{2}$-actions on the 3 -sphere

$$
S^{3}=\left\{(x, y) \in \mathbb{C} \times \mathbb{C}:|x|^{2}+|y|^{2}=1\right\}
$$

and give triangulations of the corresponding orbit spaces. By using combinatorial techniques we prove that the quotient space $S^{3} /(x, y) \sim(x, y)$ is homeomorphic to $S^{3}$. We also study restrictions of these $\mathbb{Z}_{2}$-actions on the subspace $T^{2}=\left\{(x, y) \in S^{3}:|x|=|y|\right\}$ and notice that some of the equivalent $\mathbb{Z}_{2}$-actions of the 3 -sphere are non-equivalent actions of the 2 torus $T^{2}$. We discuss topology of the orbit spaces $T^{2} / \mathbb{Z}_{2}$ and give their triangulations. In 1939 P. A. Smith proved that the set of fixed points of a periodic self-homeomorphism of the 3 -sphere is an $i$-sphere (for $i=-1,0,1$ or 2 ). The $\mathbb{Z}_{2}$-actions on $S^{3}$ are periodic homeomorphisms of period 2, so the set of their fixed points are the spheres of dimensions stated above. We give examples, in the simplicial category, of the involutions of $S^{3}$ whose fixed point sets are $S^{-1}, S^{0}, S^{1}$ and $S^{2}$ respectively.

## OP-06-1392

## Small volume link orbifolds

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2010 Mathematics Subject Classification. 57M50, 57R18
Keywords. Hypberbolic orbifolds

We will discuss lower bounds on the volume of a hyperbolic 3-orbifold whose singular locus is a link. We identify the unique smallest volume orbifold whose singular locus is a knot or link in the 3 -sphere, or more generally in a $\mathbb{Z}_{6}$ homology sphere. We will also discuss work in progress on identifying the smallest volume hyperbolic 3-orbifold whose singular locus is a link having torsion orders at least $n$ for each $n \geq 4$.

OP-06-1398

## On the Freedman's manifold $E_{8}$

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2010 Mathematics Subject Classification. 57N13
Keywords. Freedman manifold $E_{8}, \varepsilon$-mapping, simlicial complex, homotopy equivalences

In 1982 Freedman constructed compact simply connected 4-dimensional manifolds $E_{8}$ which is not simplicial complex. We are investigating $\varepsilon$-mappings of the $E_{8}$ onto finite simplicial complexes which are homotopy equivalences.

## Main Theorem.

1. For every positive number $\varepsilon$ there exists surjective $\varepsilon$-mapping $f: E_{8} \rightarrow P$ of $E_{8}$ to some finite 4-dimensional polyhedron $P$ which is homotopy equivalences.
2. There does not exist $\varepsilon$-mapping for some positive number $\varepsilon$ of the manifold $E_{8}$ onto triangulable compact 4 -dimensional manifold.

## Characteristic classes of configuration spaces and applications to discrete geometry

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2010 Mathematics Subject Classification. 55R80, 52A45
Keywords. Configuration spaces, characteristic classes, $k$-regular maps, waist of the sphere

The regular representation bundle $\xi_{d, k}$ over the configuration space $F\left(\mathbb{R}^{d}, k\right) / \mathfrak{S}_{k}$ of $k$ distinct points in $\mathbb{R}^{d}$

$$
\xi_{d, k}: \quad \mathbb{R}^{k} \rightarrow F\left(\mathbb{R}^{d}, k\right) \times_{\mathfrak{S}_{k}} \mathbb{R}^{k} \rightarrow F\left(\mathbb{R}^{d}, k\right) / \mathfrak{S}_{k}
$$

has classically been studied extensively by F. Cohen, Chisholm, Vassiliev, and many others. We will report about new computations of twisted Euler classes, Stiefel-Whitney classes and their monomials as well as corresponding Chern classes of the bundles $\xi_{d, k}$, using a variety of combinatorial and topological methods.

Thus we not only extend and complete previous work, supplying for example a proof for a conjecture by Vassiliev, but also make progress on a variety of problems from Discrete Geometry, among them
(i) the conjecture by Nandakumar and Ramana Rao that every convex polygon can be partitioned into $n$ convex parts of equal area and perimeter;
(ii) Borsuk's problem on the existence of " $k$-regular maps" $\mathbb{R}^{d} \rightarrow \mathbb{R}^{N}$ or $\mathbb{R}^{d} \rightarrow \mathbb{C}^{M}$, which are required to map any $k$ distinct points to $k$ linearly independent vectors, and
(iii) the topology underlying Gromov's "waist of the sphere theorem."

This lecture is based on joint work with Pavle Blagojevic (FU Berlin), Fred Cohen (Rochester RI), and Wolfgang Lück (Bonn).

## OP-06-1732

## Torus manifolds and toric origami manifolds

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2010 Mathematics Subject Classification. 57S15, 53D20, 14M25
Keywords. Torus action, toric origami manifold, origami template

A torus manifold is an orientable, compact, connected, smooth manifold of even dimension with an effective action of a half-dimensional torus with non-empty fixed point set. The notion of a toric origami manifold, which weakens the notion of a symplectic toric manifold, was introduced by Cannas da Silva-Guillemin-Pires and they show that toric origami manifolds bijectively correspond to origami templates via moment maps, where an origami template is a collection of Delzant polytopes with some folding data. If an orientable toric origami manifold has a fixed point, then it becomes a torus manifold. In this talk, we discuss the existence of toric origami structures on torus manifolds. We show that any simply connected torus manifold of dimension 4 can be a toric origami manifold and find torus manifolds which cannot admit toric origami structures.

## OP-06-2204

## Showing distinctness of surface links by taking satellites

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2010 Mathematics Subject Classification. 57Q45, 57Q35
Keywords. Surface link, 2-dimensional braid, chart, Roseman move, triple linking
For an oriented surface link $S$, we can take a satellite construction called a 2-dimensional braid over $S$, which is a surface link in the form of a covering over $S$. One of expected applications of the notion of a 2 -dimensional braid is that it will provide us with a method for showing the distinctness of surface links. Here we demonstrate such use for 2-dimensional braids. We investigate non-trivial examples of surface links with free abelian group of rank two, concluding that their link types are infinitely many. Our example $S_{k}$ consists of two components such that each component is of genus one. As invariants to show the distinctness, we use triple linking numbers, which are integer-valued invariants of surface links with at least three components; so we cannot use them for our case without a device. In order to overcome this situation, we take a 2 -dimensional braid over $S_{k}$ such that each component of $S_{k}$ is split into two components. Then it has four components, and we can calculate triple linking numbers. A 2-dimensional braid over a surface link is obtained from the "standard" 2 -dimensional braid by addition of braiding information. Unfortunately, if we consider the standard 2 -dimensional braid, then the triple linking is trivial. However, addition of braiding information makes a 2-dimensional braid with non-trivial triple linking, and enables us to show that $S_{k}$ and $S_{l}$ are distinct for coprime integers $k$ and $l$.

## Arc index of Kanenobu knots

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2010 Mathematics Subject Classification. 57M25, 57M27
Keywords. Arc index, Kanenobu knot, Kauffman polynomial

Every knot or link $L$ can be embedded in the union of fnitely many half planes in $\mathbb{R}^{3}$ which have a common boundary line such that each half plane intersects $L$ in a single arc. Such an embedding is called an arc presentation of $L$. The arc index of $L$ is the minimal number of half planes among all arc presentations of $L$. In this talk, we compute the arc index of Kanenobu knots. This is a joint work with Hideo Takioka.

## OP-06-2370

## Minimal $C^{1}$-diffeomorphisms of the circle which admit measurable fundamental domain

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2010 Mathematics Subject Classification. 37E15, 37C05, 37A40
Keywords. Diffeomorphism, minimality, rotation number, ergodicity
We construct, for each irrational number $\alpha$, a minimal $C^{1}$-diffeomorphism of the circle with rotation number $\alpha$ which admits a measurable fundamental domain with respect to the Lebesgue measure.

## Poster Session

## 00-06-0169

## Topological social choice model

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2010 Mathematics Subject Classification. 91B14, 55P45, 55P20
Keywords. Topological Methods, Social Choice, $n$-mean spaces, H-spaces, Eilernberg and MacLane Speces

The topological approach to social choice was developed by Graciela Chichilnisky in the beginning of the eighties, all the fundamental results about the social choice have been established by B. Eckmann in 1954 through the use of $n$-mean spaces, this presentation extensively discusses these results in a self contained way through the use of spaces of $n$-mean, groups with means and $H$-space and end with open questions relating to the main results obtained so far.

## OO-06-0217

## Quasi-metric tree in $T_{0}$-quasi-metric space

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2010 Mathematics Subject Classification. 54E35, 54E50, 05C12
Keywords. Metric interval, Metric tree, $T_{0}$-quasi-metric, Quasi-metric interval, Quasi-metric tree

In this talk, we discuss a concept of metric tree in $T_{0}$-quasi-metric spaces which we called quasi-metric tree. Comparable studies in the area of metric spaces have been conducted before by Dress [1].

## References

[1] A.W.M. Dress, Trees, tight extensions of metric spaces, and the cohomological dimension of certain groups: a note on combinatorial properties of metric spaces, Adv. in Math. 53 (1984), 321-402.
[2] O. Olela Otafudu, Quasi-metric tree in $T_{0}$-quasi-metric spaces, Topol. Appl. 160 (2013) 1794-1801.

# Tight triangulated manifolds 

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2010 Mathematics Subject Classification. 52B05, 52B22, 52B11, 57Q15
Keywords. Stacked sphere, Tight triangulation, Strongly minimal triangulation

For a field $\mathbb{F}$, a $d$-dimensional simplicial complex $X$ is called $\mathbb{F}$-tight if (i) $X$ is connected, and (ii) for all induced subcomplexes $Y$ of $X$ and for all $0 \leq j \leq d$, the morphism $H_{j}(Y ; \mathbb{F}) \rightarrow$ $H_{j}(X ; \mathbb{F})$ induced by the inclusion map $Y \hookrightarrow X$ is injective. Tight triangulations of manifolds are extremely rare, and according to a long-standing conjecture, they triangulate the given manifold with minimum number of simplices in each dimension. In this talk, we introduce some new classes of triangulated manifolds, and characterise the tight members in these classes. These result provides a uniform and conceptual tightness proof for all except two of the known tight triangulated manifolds. We show that the above conjecture holds for one of the classes. We also describe some recently discovered constructions of tight triangulations.

## OP-06-1408

# Twisted k-theory for proper actions on discrete groups 

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2010 Mathematics Subject Classification. 55N15, 55N91, 19N50, 19L41, 19K35
Keywords. Equivariant twisted K-theory, Geometric twisted K-homology, Baum Connes conjecture, Segal spectral sequence, Index theory

We present a definition of equivariant Twisted K-theory for proper actions using Fredholm bundles and discuss some of the most important properties.

This approach to twisted K-theory is very explicit in comparison with Kasparov KKtheory approach and allows to obtain some interesting properties. For example we have developed a spectral sequence whose $E_{2}$-term correspond with some cohomology with local coefficients and we have used it to obtain some explicit calculations. On the other hand using ideas of index theory and groupoids we have obtained an explicit descriptions of the some classical constructions of K-theory (Thom isomorphism, Poincare Duality and push forward maps). Finally we can also describe some geometric models of twisted K-homology and gives different descriptions of assembly maps for the Baum-Connes conjecture.

## OP-06-1771

## Minimal crystallizations of 3-manifolds

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2010 Mathematics Subject Classification. 57Q15, 57Q05, 57N10, 05C15
Keywords. Pseudotriangulations of manifolds, Crystallizations, Lens spaces, Presentations of groups.

We have introduced the weight of a group which has a presentation with number of relations is at most the number of generators. We have shown that the number of vertices in any crystallization of a connected closed 3-manifold $M$ is at least the weight of the fundamental group of $M$. This lower bound is sharp for the 3-manifolds $\mathbb{R} \mathbb{P}^{3}, L(3,1), L(5,2), S^{1} \times$ $S^{1} \times S^{1}, S^{2} \times S^{1}, S^{2} \times S^{1}$ and $S^{3} / Q_{8}$, where $Q_{8}$ is the quaternion group. Moreover, there is a unique such facet minimal crystallization in each of these seven cases. We have also constructed crystallizations of $L(k q-1, q)$ with $4(q+k-1)$ facets for $q, k \geq 2$ and $L(k q+1, q)$ with $4(q+k)$ facets for $q, k \geq 1$. By a recent result of Swartz, our crystallizations of $L(k q+1, q)$ are facet minimal when $k q+1$ are even. Our construction of a crystallization of a 3-manifold $M$ is based on a presentation of the fundamental group of $M$.

# Strong cohomological rigidity of quasitoric manifolds with second Betti number 2 

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2010 Mathematics Subject Classification. 57S25, 57R19, 14M25
Keywords. Quasitoric manifold, non-singular projective toric variety, strong cohomological rigidity
A quasitoric manifold is a $2 n$-dimensional compact smooth manifold with a locally standard $T^{n}$-action whose orbit space can be identified with a simple polytope. Hence, every nonsingular projective toric variety can be a quasitoric manifold. In this talk, we show that any cohomology ring isomorphism between two non-singular projective toric varieties (respectively, quasitoric manifolds) with second Betti number 2 is realizable by a diffeomorphism (respectively, homeomorphism).

## Relationship Between Metric Space and Topological Space

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2010 Mathematics Subject Classification.
Keywords. , (open balls in North West (G_z1), North East (G_z3) )

## OBJECTIVES OF THIS RESEARCH.

The objectives of this research are:
To construct metric and topological spaces using the Nigeria map.
To identify some metric and topological properties using the Nigeria map.
To establish some relationship between geo political zones using some topological concepts. This project work has focused on the Nigerian Map having capital cities as points in the set N as a metric space and topological space with respect to road distance as the distance function $d_{g}$. In the map of Nigeria as a space containing open balls, there are six open balls such that each open ball is from one geopolitical zone.

## RECOMMENDATIONS

This project work recommends that more resources should be expended to bridge the gap between the Northern and Southern part in terms of physical structure due to the huge difference in land mass. Also on the idea of centres of open balls in each geopolitical zone, if the government intends to position a capital project (may be a central work station) in each geopolitical zone, this project recommends the centres of the open balls of each geopolitical zones for easy access to the central work station. For instance, if the government decides to build a regional library in each geopolitical zone, for the purpose of easy access for all from their capital cities, the centres of open balls will be the best location to site the libraries.

## CONCLUSION

Metric space topology is an effective mathematical structure in analyzing any arbitrary space particularly a geographical space which this project work has done with respect to road distance from capital cities. This project has built the idea for further researches on metric space topology and geographical spaces with respect to other parameters such as are embodied in some theoretic properties of the topology.

## PO-06-0975

## 0-graphic flow

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2010 Mathematics Subject Classification. 54h20
Keywords. 0-graphic flow, minimal flow

In 1970, S. Ahmad has introduced the characteristic 0+ real flow as the closure of the orbit coincide with the first prolongation set for each point. In 1985, J. Auslander defined the graphic flow on the minimal flows and characterized the graphic flow. In this paper, we define the 0 -graphic flow which is more generalized conception than graphic flow and characterized the 0 -graphic flow.

## PO-06-1174

## On Gauss diagrams of Symmetric knots

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2010 Mathematics Subject Classification. 57M25, 57M27
Keywords.

In this talk, we will construct symmetric knots by using the method adapted from the topological graph theory, and introduce the 'voltage gauss diagram' of symmetric knot which is constucted from the information of the gauss diagram of the base knot and the corresponding group action.

PO-06-1607

## On kinoshita conjecture

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2010 Mathematics Subject Classification. 55A25
Keywords. Kinoshita conjecture, Non-oriented surface, Projective plane

The Kinoshita conjecture says that if a surface is a knotted projective plane in 4-space, then it is ambiently isotopic to the connected sum of an un-knotted projective plane and a knotted sphere. In this talk, we will give a presentation of a non-oriented surface embedded in 4 -space and, as an application, will try to find properties related to the Kinoshita conjecture.

# Fibrewise analogues of Arutyunov's theorem 

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2010 Mathematics Subject Classification. 54C05, 54C65, 54C10
Keywords. Metric mapping, map-morphism, continuous section, perfect section

The definition of metric mappings is introduced by B.A. Pasynkov (1999). For metric mappings three analogues of A.V. Arutunov's theorem on coincidence points of a pair mappings between metric spaces (one of the mappings is $\alpha$-covering, and another is $\beta$-Lipschitz) are obtained. In all three cases one considers two map-morphisms (one of them is fibrewise covering, and another is Lipschitz) of a metric fibrewise complete open mapping to the other metric mapping and both of them are mappings onto a paracompact. In the first case this paracompact is suggested 0 -dimensional, and in the third case the condition of convexity on fibres of the covering map-morphism is added. Herewith, continuous sections play role of coincidence points of mappings between metric spaces and these sections wholly consists of coincidence points of map-morphisms of metric mappings. For the second case perfect sections play such role. For proving the analogues the author used (in a more generalized version) the functional search method of T.N. Fomenko.

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PP-06-0284
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## Universal objects in some classes of free $G$-spaces

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2010 Mathematics Subject Classification. 54H15, 54D35, 54B05
Keywords. Compact Lie group, Free $G$-space, Universal object, Orbit space, Slice

The letter " $G$ " will denote a compact Lie group with the identity element $e \in G$. If in a $G$-space $X, g x \neq x$ for all $x \in X$ and $g \in G \backslash\{e\}$, then one says that that the action of $G$ on $X$ is free and $X$ is a free $G$-space.

A $G$-space $U$ is called universal for a given class of $G$-spaces $G-\mathcal{K}$, if $U \in G$-K and $U$ contains as a $G$-subspace a $G$-homeomorphic copy of any $G$-space $X$ from the class $G$ - $\mathcal{K}$.

In this talk we shall present universal $G$-spaces in the class of all paracompact (respectively, metrizable, and separable metrizable) free $G$-spaces.

Denote by Cone $G$ the cone over $G$ endowed with the natural action of $G$ induced by left translations. Let $J_{\infty}(G)$ be the infinite join $G * G * \ldots$; it is just the subset of the countable product $(\text { Cone } G)^{\infty}$ consisting of all those points $\left(t_{1} g_{1}, t_{2} g_{2}, \ldots\right)$ for which only a finite number of $t_{i} \neq 0$ and $\sum_{i=1}^{\infty} t_{i}=1$. We let $G$ act coordinate-wise on $J_{\infty}(G)$. Denote by $I$ the
unit interval $[0,1]$ and by $I^{\tau}$ the Tychonoff cube of a given infinite weight $\tau$ endowed with the trivial action of $G$.

We prove that for every infinite cardinal number $\tau$, the product $J_{\infty}(G) \times I^{\tau}$ is universal in the class of all paracompact free $G$-spaces of weight $\leq \tau$.

A similar result for metrizable free $G$-spaces of weight $\leq \tau$ is also obtained.

# A Written Proof of the Four-Colors Map Problem 

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2010 Mathematics Subject Classification. 51G05, 00A05, 54A99
Keywords. Proof, Four colors map problem, written, four colored points
A contact border of two adjacent figures can only be two adjacent borderlines. Consider the plane of any uncolored planar map as which consists of two kinds' parallel straight linear segments according to a strip of a kind alternating a strip of another, and every straight linear segment of each kind consists of two kinds of colored points according to a colored point alternating another colored point, either kind of colored points at a straight linear segment is not alike to either kind of colored points at either straight linear segment which the straight linear segment adjoins. Anyhow the plane has altogether four kinds of colored points. First we need to classify figures at an uncolored planar map and transform them. First merge orderly each figure which adjoins at most three figures and an adjacent figure which adjoins at least four figures into a figure. Secondly merge each tract of figures which adjoin at most three figures and an adjacent figure into a figure. After that, transform every borderline closed curve of figures including merging figures into a rectangular frame which has longitudinal and transversal sides. Decide a color of each figure according to a color of some points of borderlines closed curves of the figure itself. For each figure merged after every dye, must throw away dyed a color, recover original four colors' points. Then, repeat the above process from outside to inside. For each figure which adjoins at most three figures, dye a color unlike colors of adjacent figures. The paper was published at "Global Journal of Pure and Applied Mathematics" (GJPAM, ISSN 0793-1768, Online ISSN 0793-9750, Vol.9, No.1, 2013, pp.111). The full text is putting at preprint Vixra.org: http://vixra.org/pdf/1401.0130v1.pdf The GJPAM is abstracted and indexed in The Mathematical Reviews; MathSciNet; Zentralblatt Math and EBSCO databases.

# Artin braid groups and crystallographic groups 

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2010 Mathematics Subject Classification. 20F36, 20H15, 20E45, 57M99
Keywords. Artin braid groups, Crystallographic groups, Bieberbach groups

A crystallographic group of dimension $n$ is defined to be a uniform discrete subgroup of $\mathbb{R}^{n} \rtimes O(n) \subseteq \operatorname{Aff}\left(\mathbb{R}^{n}\right)$. Let $n \geq 3$, let $B_{n}$ (resp. $P_{n}$ ) denote the Artin braid group (resp. the Artin pure braid group) on $n$ strings, and let $\left[P_{n}, P_{n}\right]$ denote the commutator subgroup of $P_{n}$. In this work, we show that the quotient $\frac{B_{n}}{\left[P_{n}, P_{n}\right]}$ is a crystallographic group that has no 2-torsion. As a consequence, for a 2 -subgroup $H$ of the symmetric group $\Sigma_{n}$ the quotient $\frac{\sigma^{-1}(H)}{\left[P_{n}, P_{n}\right]}$ is a Bieberbach group (a torsion-free crystallographic group), where $\sigma_{n}: B_{n} \rightarrow$ $\Sigma_{n}$ is the canonical projection. We also present a characterization of torsion elements and conjugacy classes of the group $\frac{B_{n}}{\left[P_{n}, P_{n}\right]}$.

## NP-06-0921

# The number of isotopies of tight contact structures on the thickened hyperboilc surface 

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2010 Mathematics Subject Classification. 53D10, 57R17, 53D35
Keywords. Conact 3-manifold, tight contact structure, hyperbolic 3-manifold

In 3-dimensional contact manifolds, there is a dichotomy between tight contact structures and overtwisted contact structures. Tight contact structures are still mysterious though many tools have been developed. Among those, the number of contact isotopies of tight contact structure of mapping torus whose fiber is higher genus surface and monodromy map is pseudo-Anosov is unknown except special casees. To obtain these, we investigate the upper and lower bound of the number of contact isotopies of tight contact structures on the thinkened higher genus surface with special boundary condition using the bypass theory and sutured Floer homology.

# On equivariant extensions of differential forms for non-compact Lie groups 

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2010 Mathematics Subject Classification. 57R91, 57T10, 81T40, 81 T 70
Keywords. Equivariant cohomology, Equivariant extension, Differential form, Gauged WZW action
There are interesting relations between equivariant extensions of closed differential forms in the differentiable framework, and the existence of specific geometric structures on manifolds. Topological conditions for the existence of such equivariant extensions can be obtained whenever the group acting is Compact and of Lie type. In this talk/poster I will discuss the case on which the group acting is of Lie type but not necesarilly compact. I will show that when the group is connected, the cohomology of the equivariant Cartan complex of equivaraint differential forms surjects to the cohomology of the homotopy quotient. Therefore we can deduce that in the case of a non-compact Lie group, any closed equivariant extension of a closed differential form may be written in terms of differential forms on the equivariant Cartan complex of the non compact group. I will finish with some applications to gauged WZW actions.

Nevertheless All the previous relations hold whenever the Lie group that is acting on the manifold is compact; in this lecture I will discuss the case when the Lie group is not compact.

